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Matrix Description of M-theory on T^6

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Abstract

We give some evidence that the worldvolume theory of the M-theory KK 6-brane is governed by a non-critical membrane theory. We use this theory to give a matrix description of M-theory on T^6 .

1. Introduction

Recently there has been much excitement about a new approach to capture the non-perturbative aspects of superstring theory: matrix theory ([17]). “Matrix theory is a nonperturbative Hamiltonian formalism for the theory formerly known as string theory/M theory” ([31]). Nevertheless this nice new theory has a serious flaw: it is formulated in the infinite momentum frame (IMF), sometimes called lightcone gauge. Thus in addition to not being manifestly covariant the theory becomes background dependent. That means for every new background (e.g. for various compactifications) one needs a different Hamiltonian, ergo a new theory. Since we are mostly interested in compactifications down to four dimensions (after all, this is the real world) it is a very interesting problem to find the matrix description of M-theory compactifications. Since matrix theory is non-local, the compactified theory has ‘more’ degrees of freedom than the theory describing flat space.

Very early it has been realised that compactifications of M-theory on tori can be realised by substituting the original 0+1 dimensional quantum mechanics by $d+1$ dimensional SYM with 16 supercharges ([17,20]). This procedure works only up to T^3 . For higher dimensional tori the new degrees of freedom added by going to higher dimensional SYM are not ‘enough’. This shows up as non-renormalizability of the gauge theory. The UV of the SYM requires new degrees of freedom to be well defined. The new degrees of freedom can not be determined in any way from the IR information one has, so it is a difficult task to find the matrix description for M-theory on manifolds of dimension greater than 3.

Recently some progress has been made in this direction. Berkooz, Rozali and Seiberg ([25]) showed that the theory on M-theory 5 branes (the (2,0) fixed point) can be used to define matrix theory on the T^4 . Similarly one can obtain matrix theory on 4 manifolds breaking higher amounts of SUSY ([26]). Later Seiberg realised matrix theory on T^5 by some non-critical string theory, the theory on NS 5-branes at zero string coupling ([12]). In this work we will argue that the worldvolume theory on a KK 6-brane of M-theory is described by a non-critical membrane theory which can be used as the matrix description of M-theory on T^6 .

While finishing this work we received [32] where the authors make a similar proposal.

2. The KK-monopole

Recently the KK monopole solution of higher dimensional SUGRA theories have received renewed interest [1,2,3,4,5,6,7,8,9,10,11]. Classically these field configurations appear after compactification of at least one direction. They are the monopoles of the Kaluza Klein $U(1)$. Since U-duality transforms these monopole solutions into the various D and NS branes, it is naturally to also interpret them as some kind of brane, baptised KK brane. Since the SUSY algebras in 11 and 10 dimensions naturally contain the charges carried by these objects, it has been speculated in [4] that even the flat theory might allow these KK $(D-5)$ -brane (D being the space-time dimension).

The multiple Kaluza-Klein monopole solution is described by the metric:

$$ds^2 = -dt^2 + \sum_{m=5}^{D-1} dy^m dy^m + ds_{TN}^2,$$

where D is 10 for various string theories and 11 for M -theory, y^m denote the space-like world-volume coordinates on the $(D-5)$ -brane represented by this solution, and ds_{TN}^2 is the metric of the Euclidean multi-centered Taub-NUT space[13]:

$$ds_{TN}^2 = U^{-1}(dx^4 + \vec{\omega} \cdot d\vec{r})^2 + U d\vec{r}^2.$$

Here x^4 denotes the compact direction, and $\vec{r} \equiv (x^1, x^2, x^3)$ denotes the three spatial coordinates transverse to the brane. U and $\vec{\omega}$ are defined as follows:

$$U = 1 + \sum_{I=1}^N U_I, \quad \vec{\omega} = \sum_{I=1}^N \vec{\omega}_I,$$

where,

$$U_I = \frac{4A}{|\vec{r} - \vec{r}_I|},$$

and,

$$\vec{\nabla} \times \vec{\omega}_I = \vec{\nabla} U_I.$$

A and \vec{r}_I are parameters labelling the solution. \vec{r}_I can be interpreted as the locations of the Kaluza-Klein monopoles in the transverse space. In order that the solutions are free from conical singularities at $\vec{r} = \vec{r}_I$, x^4 must have periodicity $16\pi A$.

The radius A of the $U(1)$ part of the $SU(2) \times U(1)$ isometry of the solution corresponds to the compactification radius of M -theory, which is associated to the coupling of the resulting IIA string theory.

In what follows we will basically make use of the following two results about the KK 6-brane of M -theory and its worldvolume theory.

- The curvature of the monopole solution can be kept small everywhere by taking the compactification radius big (see e.g [8]). In the limit $A \rightarrow \infty$ ¹ the curvature goes to zero everywhere. Indeed it can be shown that by making appropriate change of coordinates one can map this to a flat metric ([14]). The KK brane decouples from gravity and since it is not charged under the three form potential it decouples from all the bulk M -theory fields. The most obvious interpretation is that there is just no brane left after decompactification.
- For finite compactification radius M -theory on a circle can be interpreted as type IIA string theory. The KK brane becomes the D6 brane. The effective worldvolume theory is 6+1 dimensional $U(N)$ SYM, where N denotes the number of coincident 6-branes

¹Remember that A denotes the radius of the eleventh dimension. We would like to reserve the letter R for the light cone direction of matrix theory.

([8,4,6]). The coupling can be determined using Polchinski's D-brane results ([15]) and turns out to be

$$\frac{1}{g^2} = \frac{M_s^3}{g_s} = \frac{1}{l_{p,11}^3}.$$

We used Wittens M-theory/IIA relations ([16]) to express everything in terms of $l_{p,11}$ and A , the radius of the 11th dimension, since in the end we want to take the limit of flat 11 dimensions. We see that in these variables the coupling of the 6 brane worldvolume theory is independent of the compactification radius (and this is only true for the 6 brane)².

Since the low-energy effective theory on the brane is totally independent of the compactification radius and hence does not see the flat limit as something special at all, a different interpretation of this flat limit might be appropriate. Let us briefly explain a proposal for the flat limit of the KK monopole. Later we will work out consequences and show that it leads to a highly consistent picture.

We take the fact that the low-energy effective theory on the worldvolume is independent of the compactification radius as an indication that in the decompactification limit the 6 brane does not disappear, but decouples from the bulk fields. That it has to decouple can be seen from the fact that the metric becomes flat and naively the brane disappears. To an 11d observer away from the brane decoupling is just as fine as disappearing. He/She doesn't feel at all that there is a brane, it might as well be gone (and hence this statement is consistent with the flat KK metric). But for the theory ON the brane it has consequences³.

The theory on the brane stays well defined. It has a scale $\frac{1}{l_{p,11}^3}$. Below this scale it is well described by 6+1 dimensional SYM. At this scale new degrees of freedom become important. For any finite compactification radius, these degrees of freedom are described by the full fledged 11d theory. In the flat 11d limit the new degrees of freedom that take over at the scale SYM breaks down are no longer the full 11d quantum theory, since the bulk fields (especially gravity) decoupled!

The conjecture thus is that in the flat 11d limit the KK 6-brane is described by a consistent 7d quantum theory decoupled from gravity.

3. Matrix Theory

3.1. The theory on the brane

Given the existence of such a theory, a natural thing to do would be to take its large N limit, where N denotes the number of coincident KK 6-branes, as a matrix description ([17]) of

²It is not clear to us if just replacing type II parameters by 11d parameters does indeed capture the full low-energy physics even at large A . We assume it does. The highly consistent picture that arises hopefully helps to justify this assumption

³One might want to worry about the coupling of the excitations on the brane to the bulk gravity. Take excitations on the KK-brane of a given energy-scale E . Since they have to preserve the isometry their energy density is constant along the circle and hence can be made arbitrarily small everywhere by taking A to be big enough. Since gravity couples to energy density, these fluctuations also decouple.

M-theory on T^6 . The consistency of the emerging picture gives some evidence to our claims. We thereby also get some further insight about the nature of the theory on the brane.

This proposal is actually dual to Seibergs proposal for matrix compactifications on the 6 torus [12]: He takes the 5 brane of a type II string theory, say IIB, at zero string coupling and takes a circle transverse to the world volume to be compact. This compact circle corresponds to one of the space-time circles, the other 5 are related to the base-space of the world volume theory on the 5 brane. Now S-dualize this setup to get a D5 brane at infinite coupling, T dualize the circle to get a IIA D6 brane at infinite coupling, now all compact directions being worldvolume directions. Now we can reinterpret the D6 brane at infinite coupling as the KK brane in flat eleven dimensions. In Seibergs analysis ([12]) the stringy excitations of the system are described by the fundamental string stuck to the NS5 brane. Under the above series of dualities this F1 string turns into the M2 brane. We hence can make our proposal more concrete: The worldvolume of the KK 6 brane in the flat 11d limit is governed by a supermembrane theory living in 7 space-time dimensions. The low-energy description of this theory is 6+1 dimensional SYM. This theory indeed contains brane like excitations, which are the instantons in 4 dimensions, become the Seiberg strings in 6 dimensions and are membranes in 7 dimensions. Note also that the classical supermembrane action is, like the Green-Schwarz string, only defined in certain dimensions. 7 is one of them ([18]). In the recent paper of Aharony, Berkooz, Kachru, Seiberg and Silverstein ([19]) they find a matrix description for this kind of theory. They discuss a M5 brane with a compact transverse 2 torus, which is, as described above, dual to what we are talking about. They remark that it is governed by a 2+1 dimensional field theory, while Seiberg's stuck string is described by a 1+1 dimensional field theory, which might be a further indication for some membrane making its appearance.

3.2. Quantitative Predictions

Since we know that the low-energy description of this fictitious 7d-membrane theory is $U(N)$ SYM, we inherit the usual SYM on the dual torus relations ([17,20]) if we use it to do matrix theory. Note that when people were using SYM on the dual torus as the matrix description of M-theory on T^6 they found quantitative agreement for all low-energy processes ([17,21]). It is thus a nice feature that our model reduces to SYM in the IR. However it was noted very early that the SYM on the dual torus description misses several degrees of freedom ([23,24,25,22]). In field theory this fact shows up as non-renormalizability. New degrees of freedom become important in the UV. We will show in the following that assuming that these additional degrees of freedom are represented by the conjectured stuck membrane theory on the KK worldvolume, one gets a correct description of M-theory on T^6 .

3.2.1. The base-space/space-time relations

We take the stuck-membrane theory living on a torus with radii Σ_i ($i=1,...,6$) and a membrane tension $\frac{1}{\lambda_p^3}$. According to the usual terminology we will refer to this space as the base-space. Certain properties of the base-space theory can be obtained by looking at the way we constructed it as the theory on a KK 6-brane in the flat limit. We will refer to this as

the auxiliary model. In the auxiliary model λ_p becomes the 11-d Planck length. This base-space theory is supposed to be the matrix description of space-time M-theory on T^6 . This space-time theory has a Planck scale L_p (which is not the same as λ_p) and radii L_i . According to the usual SYM on the dual torus description we get the following base-space/space-time relations (omitting factors of 2π):

$$\begin{aligned}\Sigma_{1,2,3,4,5,6}^B &= \frac{L_p^3}{RL_{1,2,3,4,5,6}} \\ \lambda_p^3 = g^2 &= \frac{L_p^{12}}{R^3 L_1 L_2 L_3 L_4 L_5 L_6}\end{aligned}$$

where R as usual denotes the light-cone radius of the matrix model and g is the gauge coupling of the SYM.

3.2.2. *Decompactification*

As first check let us show, that this proposal reduces correctly to Seiberg's stuck string ([12]) under decompactification of a space-time radius (which after all is the matrix description of M-theory on T^5). That is we want to take a space-time length, say L_6 to infinity. From this we see that Σ_6 goes to zero, the tension of the membrane ($\frac{1}{\lambda_p^3}$) goes to infinity while the tension of membranes wrapping Σ_6 (and hence yielding strings in the resulting 6 dimensional theory) stays constant. We are thus left with a 6 dimensional string theory with

$$\Sigma_{1,2,3,4,5}^B = \frac{L_p^3}{RL_{1,2,3,4,5}} \tag{1}$$

$$M_s^2 = \frac{\lambda_{11}^3}{\Sigma_6} = \frac{R^2 L_1 L_2 L_3 L_4 L_5}{L_p^9} \tag{2}$$

exactly as expected.

This can also be seen from the auxiliary construction. We look at KK-6 branes compactified on the base-space torus, when a circle of the base-space torus shrinks. M-theory on a vanishing circle is again type IIA, this time at zero coupling. Since we this time compactified a direction in the KK 6-brane worldvolume we turn it into a IIA KK 5-brane at zero string coupling. This is according to Sen ([8]) another realization of Seibergs string theory, as we will also discuss in the following section. The base-space to space-time relations are like those of the IIB NS 5-brane at zero coupling which are again those of the usual SYM on the dual torus. We hence get a consistent decompactification. Equivalently one could undo the series of dualities described previously to get back to type IIB NS 5-branes at zero string coupling with the radius of the transverse circle going to infinity, which is exactly Seiberg's realization of this theory.

3.2.3. *The moduli space*

According to the general philosophy of matrix theory the moduli space of the space-time theory appears as the parameter space of possible base-space compactifications. This parameter space can be obtained from the auxiliary model. Here it is obvious what we want to do:

we look at compactifications of M-theory on a seven torus in the limit that one of the circles becomes flat, which is of course the same as the moduli space of M-theory compactifications on a 6 torus, that is

$$\Gamma \backslash \frac{E_{6(6)}}{Sp(4)}$$

as expected. Note that this kind of seemingly circular logic is similar to what Seiberg had to do ([12]). We use information about M-theory on T^7 to get information about matrix theory on T^6 . We have to do this since so far the only information we have about the 7d theory is what we obtained from M-theory. But at some point we should get a better understanding of this seven dimensional theory without using M-theory. This should be much easier than studying full-fledged M-theory, since we are dealing with only 7 dimensions and decoupled gravity. Nevertheless we can see from what we already know about M-theory and hence about the non-critical membrane theory that this theory has the right moduli space. Even though this seems somewhat trivial, note that in the original form of Seiberg's conjecture for T^6 (the type II 5-brane with a compact circle, [12]) it is far from obvious how this moduli space should appear. In our formulation it is manifest.

3.2.4. *Excitations*

To study the possible excitations of the system, we would like to analyze the energy of various branes stuck to the D6 brane in our limit (decompactification of the 11th dimension).

That is, we consider a type IIA compactified on a torus with right angles and no B field present. The energy of a configuration of N D6 branes and n other branes which are all of the same type in the decompactification limit is given by ([12]) (remember that to avoid confusion with R we denote the radius of the eleventh dimension of this auxiliary model by A):

$$E = \lim_{A \rightarrow \infty} \sqrt{(N \Sigma_1 \Sigma_2 \Sigma_3 \Sigma_4 \Sigma_5 \Sigma_6 T_6)^2 + (n \Sigma_{i_1} \dots \Sigma_{i_p} T_{brane})^2} - N \Sigma_1 \Sigma_2 \Sigma_3 \Sigma_4 \Sigma_5 \Sigma_6 T_6$$

where T_6 denotes the tension of the D6 and T_{brane} the tension of the other p-brane. We should express T_6 and T_{brane} in 11d quantities since we want to analyze the limit where A goes to infinity at fixed 11d Planck scale λ_p . In this limit most branes become degenerate with the D6 brane, that is for them the above expression vanishes. To see which branes give a finite contribution we calculated the tensions of all IIA branes that can be completely wrapped on the 6 torus (that is everything except for the D8).

Brane	$\frac{1}{g^2}$	T_{brane}
D0	$\frac{l_s^3}{g_s} = \frac{\lambda_p^6}{A^3}$	$\frac{1}{l_s g_s} = \frac{1}{A}$
D2	$\frac{l_s}{g_s} = \frac{\lambda_p^3}{A^2}$	$\frac{1}{l_s^3 g_s} = \frac{1}{\lambda_p^3}$
D4	$\frac{1}{l_s g_s} = \frac{1}{A}$	$\frac{1}{l_s^5 g_s} = \frac{A}{\lambda_p^6}$
D6	$\frac{1}{l_s^3 g_s} = \frac{1}{\lambda_p^3}$	$\frac{1}{l_s^7 g_s} = \frac{A^2}{\lambda_p^9}$
F1		$\frac{1}{l_s^2} = \frac{A}{\lambda_p^3}$
NS5	$\frac{1}{l_s^2} = \frac{A}{\lambda_p^3}$	$\frac{1}{l_s^6 g_s^2} = \frac{1}{\lambda_p^6}$
KK5		$\frac{1}{l_s^7 g_s} = \frac{\Sigma_i^2}{\lambda_p^9}$

As above $\frac{1}{g^2}$ denotes the worldvolume gauge coupling, A the radius of the 11th dimension, l_s the string length, g_s the string coupling and λ_p the 11d Planck length of our auxiliary model. To convert to 11d units we used Witten's formulas ([16]): $l_s = \frac{\lambda_p^{3/2}}{A^{1/2}}$, $g_s = \frac{A^{3/2}}{\lambda_p^{3/2}}$. To get the tension of the KK5 one has to consider how this one arises. We take a KK 6-brane of M-theory and wrap it around A . Its worldvolume is hence transverse to one of the circles, say Σ_i . It thus has the same tension as the D6 with the role of A and Σ_i interchanged. While Σ_i now determines the tension as indicated above, A appears as one of the wrapped circles in the energy formula.

Inspection of this table shows us that the above energy formula gives us zero for most of the branes. The only branes yielding a finite contribution are the F1, the D4 and the KK5. The resulting energies are

$$\begin{aligned}
E_i^E &= \frac{n^2 \Sigma_i^2 \lambda_p^3}{2N \Sigma_1 \Sigma_2 \Sigma_3 \Sigma_4 \Sigma_5 \Sigma_6} \\
E_{ij}^M &= \frac{n^2 \Sigma_1 \Sigma_2 \Sigma_3 \Sigma_4 \Sigma_5 \Sigma_6}{2N \lambda_p^3 (\Sigma_i \Sigma_j)^2} \\
E^i &= \frac{n^2 \Sigma_i^2 \Sigma_1 \Sigma_2 \Sigma_3 \Sigma_4 \Sigma_5 \Sigma_6}{2N \lambda_p^9}
\end{aligned}$$

The indices E and M denote that these energies are seen as electric and magnetic fluxes in the low-energy SYM. The low-energy description misses the states corresponding to the KK5. These excitations naturally combine themselves into a 27 of the discrete $E_{6(6)}$ duality group. Using the base-space/space-time relations and converting to space-time masses one sees that these excitations correspond to BPS particles with masses:

$$\begin{aligned}
M_i^E &= \frac{1}{L_i} \\
M_{ij}^M &= \frac{L_i L_j}{L_p^3} \\
M_i &= \frac{L_j L_k L_l L_m L_n}{L_p^6}
\end{aligned}$$

where i, j, k, l, m, n are a permutation of 1,2,3,4,5,6. These energies correspond exactly to those expected from the M-theory compactification, 6 momentum modes 15 wrapped membranes and 6 wrapped fivebranes.

3.2.5. The momentum multiplet

In addition to these finite energy BPS states the matrix model also has states which correspond in space-time to objects wrapping the lightcone direction R . Since the matrix model procedure [17] tells us to take R to infinity, these states are moved off to infinite energy in the end. Nevertheless it was the membrane wrapping R and one of the space-time circles which allowed [27] to indentify the base-space and space-time variables according to what we called the usual SYM on the dual torus procedure. In [28] the authors baptised this kind of multiplet the momentum multiplet, since all the states can be obtained by U-duality acting on base-space momentum modes.

For the $d = 6$ case we are interested in they find that the matrix description should contain the following states:

E_{SYM}	$M_{spacetime}$
$\frac{1}{\Sigma_i}$	$\frac{RR_i}{L_p^3}$
$\frac{\Sigma_i \Sigma_j}{\lambda_p^3}$	$\frac{RR_i R_j R_k R_l}{L_p^6}$
$\frac{\Sigma_i \Sigma_j \Sigma_k \Sigma_l \Sigma_m}{\lambda_p^6}$	$\frac{VR_i R}{L_p^9}$

where $V = R_1 R_2 R_3 R_4 R_5 R_6$. In space-time these objects obviously correspond to membranes, 5-branes and 6-branes wrapping R (the 6 brane wraps R and 5 of the space-time circles, its tension is given by $\frac{R^2}{L_p^9}$). Together with the 27 BPS excitations from above, the momentum mode around R and the 6-brane with tension $\frac{R}{L_p^9}$ wrapping all 6 space-time circles, these states form a 56 of E_7 , which is the full expected U-duality group for finite R , where this matrix model really describes M-theory on T^7 . A related discussion can be found in [29].

[28] were able to identify the first two entries in the low-energy SYM description of the matrix model. The first correspond to momentum modes along the compact base-space directions. The second one is the wrapped Yang-Mills instanton, which is a $(d-4)$ brane for SYM on T^d . In our case it is just our non-critical membrane! This is possible since in the flat $A \rightarrow \infty$ limit we were taking to decouple gravity, the membrane tension is independent of A and hence the wrapped membrane yields a finite energy state (for finite R). This in fact allowed us to treat the membrane as the fundamental excitation of the full 7d theory. If we now ask ourselves which other objects have finite tension in the $A \rightarrow \infty$ limit, a short inspection of the table with the tensions we worked out previously shows that there is precisely one more such object, the NS 5-brane with a tension of $\frac{1}{\lambda_p^6}$. The wrapped 5 brane gives us the 6 missing states with the right energy! All these objects transform into each other under U-dualities. The fundamental U-duality invariant object [28] identified as $\frac{\Sigma_1 \Sigma_2 \Sigma_3 \Sigma_4 \Sigma_5 \Sigma_6}{\lambda_p^9}$ can be identified as the product of 5-brane tension, membrane tension and base-space volume.

4. Remarks

In this work we conjectured the existence of a non-critical membrane theory living in 7 space-time dimensions. The existence of such a theory would lead to a unification of the various 6 dimensional non-critical string [12,30] theories like M-theory did for the various string theories in 10 dimensions. At some points of its moduli space the compactified membrane theory is best described by some non-critical string theory. We already saw that compactification on a circle gives us back Seiberg's 6 dimensional type II stuck strings. Similar one would expect that compactification on an interval leads to the stuck type I.

If this story is right it would mean that to incorporate the 5 brane in matrix theory, we should not only include the particles charged electrically under the KK $U(1)$, which are the zero branes, but also the dual objects charged magnetically under the $U(1)$, the KK monopoles. This might be a sign of how the principle of duality becomes manifested in the matrix theory.

There is an obvious generalization to the work done so far. If one wants to consider M-theory compactifications on 6 manifolds preserving less supersymmetry ($K3 \times T^2$ or CY), the matrix description of this should obviously be the non-critical membrane theory described above living on a corresponding manifold, breaking the same fraction of supersymmetry.

Remains the question how to describe compactifications on even higher dimensional tori. So far we managed to compactify down to 5 dimensions. But we definitely want to understand compactifications to 4 dimensions! There is one more Kaluza Klein like object in M-theory, the conjectured M9 brane. We call it Kaluza Klein like ([33]), since it naively only appears in compactifications ([34]), like the KK monopole. For example double dimensional reduction of this M9 brane is supposed to yield the D8 brane of IIA. From this we see that the worldvolume of this M9 brane is 10d SYM ([4]). Like for the KK 6-brane, the SUSY algebra of 11d SUGRA contains a central charge for a 9-brane, so it was speculated that there might actually be a 9-brane solution in uncompactified M-theory. Since the D8 only is possible in a massive IIA background and there is no known generalization of the massive type IIA to eleven dimensions, it is not even possible to construct the 9-brane as a classical SUGRA solution.

After the experiences we made with the KK 6-brane one can nevertheless hope for a similar mechanism taking place. Instead of disappearing the worldvolume theory of the M9 brane might decouple from the bulk fields in the flat 11d limit, giving rise to a well defined 10 dimensional theory without gravity. The gauge coupling of the SYM on this hypothetical brane can be obtained from the gauge coupling of the D8 which is:

$$\frac{1}{g_8^2} = \frac{1}{l_s^5 g_s} = \frac{A}{l_p^6}.$$

Since the SYM on the M9 brane should reduce to this after dimensional reduction on the circle with radius A , its coupling would be

$$\frac{1}{g_9^2} = \frac{1}{g_8^2 A} = \frac{1}{l_p^6}$$

again independent of the radius of the eleventh dimension! If existent this theory could help us to formulate the matrix theory of M-theory compactifications on higher tori.

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- [33] P. S. Howe, N. D. Lambert, P. C. West, hep-th/9707139.
- [34] E. Bergshoeff, M. de Roo, M. B. Green, G. Papadopoulos, P. K. Townsend, hep-th/9601150